Micro C Winter Exam 1 - Solution

PLEASE ANSWER ALL QUESTIONS.
PLEASE EXPLAIN YOUR ANSWERS.

1. **Nash Equilibrium and Subgame-Perfect Nash Equilibrium.**

(a) Find all the pure and mixed Nash Equilibria of the following game.

		Player 2		
		t_1	t_2	t_3
Player 1	s_1	2, 0	3, 2	1, 6
	s_2	3, 3	2, 1	0, 2

Solution: There are two pure-strategy NE: (s_2, t_1) and (s_1, t_3). For the mixed-strategy equilibrium, let P1's strategy be denoted $(p, 1-p)$ and P2's be denoted $(q_1, q_2, 1-q_1-q_2)$. Notice that t_2 is strictly dominated by t_3, so in equilibrium $q_2 = 0$.
Thus, the players are indifferent between their (non-dominated strategies) when

$$q_1(2) + (1-q_1)(1) = q_1(3) + (1-q_1)(0) \Leftrightarrow q_1 = 1/2$$
$$p(0) + (1-p)(3) = p(6) + (1-p)(2) \Leftrightarrow p = 1/7.$$

So the mixed-strategy NE is $(p; q_1, q_2) = (1/7; 1/2, 0)$.

□

(b) Suppose now that we introduce a new strategy for Player 1. Denote the corresponding game by G:

		Player 2		
		t_1	t_2	t_3
	s_1	2, 0	3, 2	1, 6
Player 1	s_2	3, 3	2, 1	0, 2
	s_3	1, 4	10, 10	0, 12

Use iterated elimination of strictly dominated strategies to simplify the game. Explain briefly each step (1 sentence). What is the set of Nash Equilibria of G? (*Hint*: No new calculations are required.)

Solution: Again, t_2 is strictly dominated by t_3. After eliminating t_2, then s_3 is strictly dominated by s_1. After eliminating s_3, no strategies are strictly dominated. This game is equal to the game in (a) after eliminating the strictly dominated strategy t_2. Hence, the set of NE is the same in the two games.

□

(c) Now suppose we repeat G twice. Denote the resulting game by $G(2)$. How many proper subgames are there (not counting the game itself)? Show that there is a Subgame-perfect Nash Equilibrium of $G(2)$ in which (s_3, t_2) is played in stage 1.

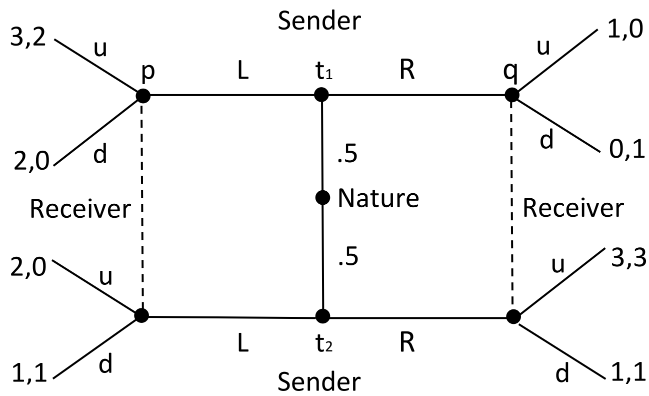
Solution: One proper subgame after each possible outcome in G: 9 proper subgames.

Proposed equilibrium strategies: in stage 1, play (s_3, t_2); in stage 2, play (s_1, t_3) on the equilibrium path and (s_2, t_1) off the equilibrium path.

Check deviations: In stage 2, a NE is played in each subgame, so no profitable deviations. In stage 1, P1 gets $10+1=11$ on the equilibrium path, and at most $3+3<11$ from a deviation. P2 gets $10+6=16$ on the equilibrium path, and at most $12+3<16$ from a deviation. Hence, the proposed equilibrium strategies form a SPNE.

□

2. **Signaling.** Consider the following signaling game.



(a) Find all the (pure strategy) separating Perfect Bayesian Equilibria (PBE).
Solution: $(LR, uu; p=1, q=0)$ is the unique separating PBE.
Case 1. Suppose $m(t_1)=L$ and $m(t_2)=R$. Then $p=1$ and $q=0$. Thus, $a(L)=u$ and $a(R)=u$. Can check that $u_S(L,u;t_1)\geq u_S(R,u;t_1)$ and $u_S(R,u;t_2)\geq u_S(L,u;t_2)$ hold. Hence: PBE.
Case 2. Suppose $m(t_1)=R$ and $m(t_2)=L$. Then $p=0$ and $q=1$. Thus, $a(L)=d$ and $a(R)=d$. Can verify that $u_S(R,d;t_1)<u_S(L,d;t_1)$. Hence, not a PBE.

□

(b) Find the (pure strategy) pooling equilibrium in which both types send message L. Does it satisfy signaling requirement 5 (SR5)? Explain briefly.
Solution: Suppose $m(t_1)=m(t_2)=L$. Then $a(L)=u$ (since $\frac{1}{2}(2)+\frac{1}{2}(0)>\frac{1}{2}(0)+\frac{1}{2}(1)$). Check sender's incentives: $u_S(L,u;t_1)\geq u_S(R,a(R);t_1)$ for all $a(R)$ whereas $u_S(L,u;t_2)\geq u_S(R,a(R);t_2)$ only if $a(R)=d$. It is optimal for the receiver to choose $a(R)=d$ if

$$q(1)+(1-q)(1)\geq q(0)+(1-q)(3)\Leftrightarrow q\geq 2/3.$$

Thus: $(LL, ud; p=1/2, q\geq 2/3)$ is a pooling PBE.
Notice that R is strictly dominated by L for t_1, but not for t_2. Therefore, SR5 prescribes that $q=0$. Hence, the pooling PBE we just found does not satisfy SR5.

□

(c) Suppose you are a politician and you want to prove that you are trustworthy and uncorruptible.

i. Give an example of a signal that is not credible and explain briefly (1 sentence) why it is not credible.

ii. Give an example of a signal that is credible and explain briefly (1 sentence) why it is credible.

Solution: A non-credible signal could for instance be 'cheap talk': both kinds of politicians would be expected to claim to be trustworthy. A credible signal could be to introduce tougher anti-corruption legislation and publishing one's own financial information, both of which are supposedly more costly for a corrupt politician.

□

3. **Coalitions.** Three entrepreneurs are considering starting a new tech company. They are free to form a company of any size between themselves. Entrepreneurs A and B are very experienced, with A being slightly more experienced than B, whereas entrepreneur C has no experience whatsoever. If entrepreneurs A and B work together in the company, the value of the company is 5000 gazillion dollars (regardless of whether entrepreneur C joins the company). If entrepreneur A starts the company alone or with C, it is worth 2000 gazillion dollars. If entrepreneur B starts the company alone or with C, it is worth 1000 gazillion dollars. If entrepreneur C starts the company alone, it is worth 0 gazillion dollars.

   (a) Think of this situation as a coalitional game with transferable payoffs. Write down the value of each coalition.
   Solution: The values are:

$$\begin{aligned} V(ABC) = V(AB) &= 5000, \\ V(AC) = V(A) &= 2000, \\ V(BC) = V(B) &= 1000, \\ V(C) &= 0. \end{aligned}$$

□

   (b) Find the core of this game.
   Solution: The coalition values give us the following restrictions:

$$\begin{aligned} V_A + V_B &\geq 5000, \\ V_A &\geq 2000, \\ V_B &\geq 1000, \\ V_C &\geq 0. \end{aligned}$$

   Hence, the core is equal to $\{(V_A, V_B, V_C) = (2000 + v, 3000 - v, 0), v \in [0, 2000]\}$.

□

4. **Spence education model.** Consider the following version of Spence's education signaling model, where a firm is hiring a worker. Workers are characterized by their type θ, which measures their ability. There are two worker types: $\theta \in \{\theta_L, \theta_H\}$. Nature chooses the worker's type, with $p_H = \mathbb{P}(\theta = \theta_H)$ and $p_L = \mathbb{P}(\theta = \theta_L) = 1 - p_H$. The worker observes his own type, but the firm does not.

   The worker can choose his level of education: $e \in \mathbb{R}^+$. The cost to him of acquiring this education is
$$c_\theta(e) = 2 \cdot \frac{e}{\theta}.$$
   Education is observed by the firm, who then forms beliefs about the worker's type: $\mu(\theta|e)$. We assume that the marginal productivity of a worker is equal to his ability and that the firm is in competition such that it pays the marginal productivity: $w(e) = \mathbb{E}(\theta|e)$. Thus, the payoff to a worker conditional on his type and education is

$$u_\theta(e) = w(e) - c_\theta(e).$$

   Suppose for this exercise that $\theta_H = 3$ and $\theta_L = 1$.

(a) In a separating equilibrium the low-ability worker chooses education level e_L and obtains wage $w_L = w(e_L)$. Is it possible that $e_L > 0$? Explain briefly (max. 3 sentences).

Solution: No. Suppose $e_L > 0$. In a separating equilibrium, θ_L gets $1 - e_L$. For any beliefs we have $u_L(0) = w(0) \geq E(\theta_L) = 1 > 1 - e_L$: profitable deviation.

□

(b) Find a separating pure strategy Perfect Bayesian Equilibrium where the two types choose education levels e_L and e_H, respectively, and the low ability type is indifferent between choosing e_L and e_H. Assume that off the equilibrium path, the firm assigns zero probability to the worker being type θ_H.

Solution: By assumption, $\mu(\theta_H|e)$ is equal to 1 if $e = e_H$ and equal to 0 otherwise. Thus, $w(e)$ is equal to 3 when $e = e_H$ and equal to 1 otherwise. We argued above that $e_L = 0$ in equilibrium. Given $w(e)$, $e = e_L = 0$ strictly dominates all $e \neq e_H$ for both types. Hence, only the strategies e_L and e_H need to be considered. To make the low type indifferent:

$$u_L(0) = u_L(e_H) \Leftrightarrow 1 = 3 - \frac{2e_H}{1} \Leftrightarrow e_H = 1.$$

Clearly, the high type prefers $e_H = 1$ as

$$u_H(e_H) \geq u_H(0) \Leftrightarrow 3 - \frac{2e_H}{3} = \frac{7}{3} \geq 1$$

holds. Hence: the specified $w(e)$ and $\mu(\cdot|e)$, together with $e_L = 0$ and $e_H = 1$ form a PBE.

□

(c) Let $p = p_H$. Find a pooling pure strategy Perfect Bayesian Equilibrium in which both types choose education level $\bar{e}$, and the low ability type is indifferent between choosing $e = 0$ and $e = \bar{e}$. Assume that off the equilibrium path, the firm assigns zero probability to the worker being type θ_H. Does the pooling equilibrium of (c) satisfy SR6? You can show this either graphically or algebraically.

Solution: By assumption, $\mu(\theta_H|e)$ is equal to p if $e = \bar{e}$ and equal to 0 otherwise. Thus, $w(e)$ is equal to $p(3) + (1-p)(1) = 1 + 2p$ when $e = \bar{e}$ and equal to 1 otherwise. Given $w(e)$, $e = 0$ strictly dominates all $e \neq \bar{e}$ for both types. Hence, only these two strategies need to be considered. To make the low type indifferent:

$$u_L(0) = u_L(\bar{e}) \Leftrightarrow 1 = 1 + 2p - \frac{2\bar{e}}{1} \Leftrightarrow \bar{e} = p.$$

Clearly, the high type prefers $e = \bar{e} = p$ over $e = 0$ as

$$u_H(\bar{e}) \geq u_H(0) \Leftrightarrow 1 + 2p - \frac{2\bar{e}}{3} = 1 + \frac{4p}{3} \geq 1$$

holds. Hence: the specified $w(e)$, $\mu(\cdot|e)$, together with $\bar{e} = p$ form a PBE.

Checking SR6: For the low-ability type, the equilibrium strategy strictly dominates e whenever

$$1 + 2p - \frac{2\bar{e}}{1} > 3 - \frac{2e}{1} \Leftrightarrow e > 1.$$

For the high-ability type, the equilibrium strategy strictly dominates e whenever

$$1 + 2p - \frac{2\bar{e}}{3} > 3 - \frac{2e}{3} \Leftrightarrow e > 3 - 2p.$$

Hence, $e \in (1, 3 - 2p)$ are equilibrium dominated for θ_L but not for θ_H. SR6: $\mu(\theta_H|e) = 1$ and hence $w(e) = 3$ for $e \in (1, 3-2p)$. The pooling equilibrium does not satisfy SR6.